

For Wednesday

- No reading
- Research paper topic due on Blackboard
- Homework:
 - Chapter 9, exercises 11, 15, 16

Exam 2

- Friday
- Take home portion handed out Wednesday, due Monday.

Program 4

- Any questions?

Research Paper

- Any questions?

Network Flow

- We have a weighted directed graph with a **source** and a **sink**--a **network**.
- Note that cycles are possible.
- Each edge's weight represent the maximum possible **flow** through that edge.
- We want to determine the maximum possible flow through the network and a way to achieve that maximum.

Ford-Fulkerson Method

- Start with 0 flow in the network
- Repeat
 - Select a path from source to sink with no full **forward** edges or a non-full **backward** edge.
 - Increase flow by maximum possible amount on that path
- Until no paths can be selected
- You now have maximal flow

Selecting a Path to Augment

- If you select the longest path, problems can arise.
- If the shortest available path is used at every iteration: the number of paths used before maximum flow is found is less than VE
- Can be improved by using the path with the best improvement to flow at each iteration.

Minimum Spanning Tree

- Useful to solve problems like: what's the cheapest way to connect a set of points to each other
- Can be solved using a greedy algorithm
- So we can use best-first search (search using a priority queue)
- Applies to undirected graphs

Minimum Spanning Trees

- Prim's algorithm

Select a node to put in the spanning tree
while some nodes are not in the tree

 select the shortest edge connecting a node
 in the tree to a node NOT in the tree

 add that edge and node to the tree

end while

Minimum Spanning Trees

- Kruskal's Algorithm

while not all nodes are in a single tree

 select the shortest edge with at least one

 end node not currently in a tree or

 connecting to trees to each other

 add that edge and its end nodes to the tree

end while

Critical Path

- The longest path between two nodes
- For what kind of graph is this meaningful?
- Why would we want to compute it?
- How could we compute it?

Transitive Closure

- The problem is: for each node in the graph, what other nodes can be reached from that node?
- Not an issue in an undirected graph. Why?
- Can be done by doing a search from each node and discovering all of the nodes that can be found from each node.

Transitive Closure (cont.)

- Depth First Search can be used to compute the transitive closure of a graph represented with an adjacency list in $O(V(V+E))$ time.
- DFS can be used to compute the transitive closure of a graph represented with an adjacency matrix in $O(V^3)$ time.

Warshall's Algorithm

- Note that if there is a path from X to Y and there is a path from Y to Z , then there is a path from X to Z
- for ($y = 0; y < V; y++$)
 for ($x = 0; x < V; x++$)
 if ($a[x][y]$)
 for ($j = 0; j < V; j++$)
 if ($a[y][j]$)
 $a[x][j] = 1;$

All Shortest Paths

- In sparse matrix, just run Dykstra's algorithm for each vertex
- In dense graphs, use a matrix and use an algorithm similar to Warshall's algorithm
- Computes all shortest paths in $O(V^3)$ time
- To determine actual path, need an additional matrix.

Floyd's Algorithm

- for ($y = 0; y < V; y++$)
 for ($x = 0; x < V; x++$)
 if ($a[x][y]$)
 for ($j = 0; j < V; j++$)
 if ($a[y][j] > 0$)
 if ($!a[x][j] \parallel$
 ($a[x][y] + a[y][j] < a[x][j]$))
 $a[x][j] = a[x][y] + a[y][j];$